The Influence of Spatial Variations of Diffusion Length on Charge Collected by Diffusion from Ion Tracks

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***Abstract*— Charge collected by diffusion from ion tracks in a semiconductor substrate may be influenced by the substrate diffusion length, which is related to recombination losses. A nonuniform spatial distribution of recombination centers results in a nonuniform diffusion length function. A theoretical analysis shows that, excluding some extreme cases, charge collection is insensitive to spatial variations in the diffusion length function, so it is possible to define an effective diffusion length having the property that collected charge can be approximated by assuming a uniform diffusion length equal to this effective value. Extreme cases that must be excluded are those in which a large number of recombination centers are confined to a narrow region near the substrate boundary.**

***Index Terms*—Charge collection, diffusion, diffusion length, ion track, recombination.**

I. Introduction

THE SUBJECT considered is charge collection in a silicon device by diffusion from an ionizing source when collected charge is limited by recombination losses in the substrate. The recombination loss is represented by a finite diffusion length in the diffusion equation. A nonuniform spatial distribution of recombination centers (RC's) results in a nonuniform diffusion length function. The specific question considered is whether collected charge is sensitive or insensitive to variations of the diffusion length function in the vertical coordinate. A theoretical analysis shows that charge collection is insensitive to such variations, so, for all but the most obstinate cases, it is possible to define an effective diffusion length having the property that collected charge can be roughly approximated by assuming a uniform diffusion length equal to this effective value.

The analysis and conclusions apply to a number of arrangements (e.g., solar cells having reduced lifetimes due to displacement damage), but the prototype arrangement assumed for discussion is an ion track which extends into a heavily doped substrate below an epitaxial (epi) layer in a large-area silicon epi diode. Computer simulations have shown [1] that charge collection includes not only charge liberated in the epi layer, but also charge liberated in the heavily doped substrate below. Charge flow from the substrate to the epi layer is strongly influenced by recombination losses because the carrier lifetime is very short in the heavily doped substrate. Computer simulations have also shown [2] that the minority carrier diffusion equation, which contains the minority carrier diffusion length, provides a rough approximation for charge reaching the epi layer from the substrate as a function of time. The track density can exceed the doping density even in a heavily doped substrate, but the four-dimensional space–time volume characterized by this condition is sufficiently small for this equation to provide a rough approximation. Although the approximation is rough for charge reaching the epi as a function of time, simulations show [2] that the minority carrier diffusion equation is very accurate for calculating total (integrated in time from zero to infinity) charge reaching the epi, which is added to the charge liberated within the epi to produce total collected charge. Simulations show that this accuracy is very good whether the ion linear energy transfer (LET) is 1 or 40 MeV-cm^2/mg. Charge collection estimates from such calculations were also found to fit experimental charge collection measurements very accurately [3].

That the diffusion equation describes total charge collection in epi diodes has been demonstrated both experimentally and by computer simulations in earlier papers as discussed above. Hence relevancy of this equation has already been established. The present paper considers mathematical approximations for solving the diffusion equation to obtain estimates of collected charge. This equation is only useful if it simplifies calculations (otherwise, we may as well let a computer simulation solve the charge collection problem). A complication is that the diffusion length is not likely to be spatially uniform because the RC distribution is not likely to be uniform. If it is necessary to account for a nonuniform diffusion length, the diffusion equation is no longer useful because calculations are no longer simple. The objective of the present paper is to show that, excluding some extreme cases, reasonably accurate charge collection estimates can be obtained from the "uniform approximation" which assumes that the diffusion length is spatially uniform and equal to some appropriately selected effective value. In other words, because applicability of the diffusion equation has already been established, the objective is merely to show that a simple analytical calculation (the uniform approximation) produces nearly the same answer as

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a more difficult analytical calculation (the diffusion equation with a nonuniform diffusion length) over a broad range of conditions. Therefore, the theory is purely mathematical. Unfortunately, this broad range of conditions does not include all conditions. There are some problem cases in which the uniform approximation fails badly. Therefore, a second objective is to identify the "problem cases" so that the reader will know when the uniform approximation can be used and when it should not be used. Fortunately, these problem cases are so extreme that they should rarely, if ever, occur in the real world.

The analysis will show that charge collection is insensitive to spatial variations in the RC distribution, even when these variations are large. To make this statement more precise, suppose two RC distributions have the property that both give the same collected charge for an effectively infinitely long track. If we now select an arbitrary track, long or short, and compare the two distributions with the same track in both, we will find that, excluding the problem cases, the two distributions also give approximately the same collected charge for this track. In particular, if we are given some actual RC distribution and then construct a uniform distribution with the density selected to give the same collected charge for an infinitely long track as the given distribution, this uniform distribution will also approximate the actual distribution, excluding the problem cases, with regards to charge collection from any track, long or short. Therefore, the effective value for the diffusion length that should be used with the uniform approximation is the value that makes the uniform approximation become exact for tracks that are effectively infinitely long. This same effective value can be used with the uniform approximation to obtain reasonably good charge-collection estimates (excluding the problem cases) for any track length, long or short.

Although a prototype arrangement (an ion track extending into a heavily doped substrate below an epi layer in a silicon epi diode) is selected for visualization and to provide terminology (allowing statements to be expressed in terms of physical quantities instead of mathematical abstractions), the analysis to follow is merely a mathematical investigation of the diffusion equation and may also apply to some other device types and/or materials. However, when considering physical arrangements other than this prototype, it should be noted that there are two separate questions. The first is whether the diffusion equation applies, and the second is, given that the diffusion equation does apply, does the uniform approximation apply. The sections to follow focus on the second question. The first question was discussed here only for the prototype arrangement. It is up to the reader to find the answer to the first question for other physical arrangements of interest to the reader. However, some guidance might be found in earlier work emphasizing that the diffusion equation may have other applications. For example, Wouters [4] used diffusion theory to predict the performance of some low-voltage radiation detectors. On the subject of single event upsets (SEU), Dodd *et al.* [5] concluded that multiple-bit SEU in at least some double static random access memory (SRAM) cell structures from "between-node" strikes is by diffusion. Zoutendyk *et al.* [6] postulated that charge collection by remote nodes in DRAM's leading to multiple-bit SEU is by diffusion. Moreau *et al.* [7] concluded that diffusion results in "extended sensitive areas" for SEU in complementary metal–oxide–semiconductor SRAM's. Smith *et al.* [8] postulated that diffusion may explain SEU cross section features better than drift for small devices with long time constants. A more recent paper [9] concluded that diffusion theory predictions agree with measured SEU cross section versus LET curves for many devices, if the LET is large enough for the cross section to be large enough so that ion hits at the cross-section perimeter require that charge first be transported by diffusion before reaching the sensitive node. On a more academic level, two earlier papers [2], [10] concluded that even when funneling is strong, there is still an interrelationship between drift and diffusion so that charge collection is intimately related to diffusion, and it is necessary to solve the diffusion equation (among other things) to calculate collected charge.

Another issue worth mentioning is that several of the papers referenced in the above paragraph [2], [4], [8] include recombination losses in their analytical results, but a spatially uniform diffusion length is always assumed. By arguing that this assumption is fine (i.e., answers are approximately correct even when the assumption is wrong), the present paper enhances the value of some of this earlier work.

II. A Preview of Conclusions to Follow

The motivation behind a set of mathematical arguments, leading to a set of conclusions, is clearer when it is known in advance what the conclusions will be (i.e., a proof is easier to follow if we first state the theorem that is to be proven). Therefore, a preview of the conclusions to follow is given here.

The problem considered is the prototype arrangement previously described subject to the following qualifications. The RC density is assumed to be uniform in the lateral coordinates but can be nonuniform in the vertical coordinate. This density is assumed to be increasing with depth up to some peak value at some arbitrary depth and then decreasing with depth, but is an otherwise arbitrary function of depth. This condition is expected to include most cases of practical interest in which the RC distribution is created by displacement damage. The track must extend from the top of the substrate (or higher, but the track section above the substrate is not relevant to this discussion) to some depth below, i.e., we exclude tracks, such as might be produced by proton-induced nuclear reaction products, which begin and end in the substrate interior. There may be a limitation regarding longitudinal track structure, but this discussion is deferred to the end of this section. The track radius is arbitrary. The objective is to determine whether the uniform approximation applies and to obtain an estimate of the effective diffusion length to be used in this approximation.

We start with the effective diffusion length estimate. This is the value that makes the uniform approximation exact when predicting collected charge from tracks that are infinitely long. One experimental method for finding this value is to measure collected charge [3] from long ion tracks having a nearly constant LET over most of the track length. The track is long enough if significant changes in track length do not significantly change the ratio of collected charge divided by ion LET. The effective diffusion length is the charge diffusing to the epi layer (which is the total collected charge minus the charge liberated in the epi layer) divided by the linear track density (charge per unit length which is calculated from the ion LET).

To determine applicability of the uniform approximation, it is necessary to estimate the location of the peak in the RC density. This requires some knowledge of how the RC distribution was created. If it is due to displacement damage from highly penetrating particles, no estimate is needed because the uniform approximation can be taken for granted. If the RC distribution is due to displacement damage from particles that stop in the device, and if it can be assumed that most of the damage is at the end of the particle range, the stopping depth is an estimate of the location of the peak in the RC density.

The conclusions are as follows: if the depth at which the RC density is greatest is less than 3/2 of the effective diffusion length, do not use the uniform approximation. This could be (probably not, but possibly) a problem case in which the approximation fails badly. If this depth is greater than or equal to 3/2 of the effective diffusion length, then the uniform approximation is at least reasonably accurate. Errors between predicted and actual charge collected from a given track are on the order of 20% or less, regardless of how uniform the RC density is and regardless of track length. Whether the approximation is merely reasonably accurate or is excellent for arbitrary track lengths depends on how uniform the RC density is. However, even when the RC density varies as much as can be seen by looking ahead to Fig. 6(a) (which plots the reciprocal of the diffusion length against depth), the accuracy is quite good, as shown in Fig. 6(b) (which compares plots of a normalized collected charge versus track length). RC density variations must be very extreme in order for the uniform approximation to not be good.

It should be acknowledged that there may be a limitation regarding longitudinal track structure. The analysis used to derive the conclusions applies to a linear track density that is uniform over a finite track length that begins at the top of the substrate and ends at some arbitrary depth. However, a superposition of such tracks can simulate any nonuniform linear density that does not increase with depth. Therefore, if the uniform approximation is accurate for uniform tracks starting at the top of the substrate and having arbitrary length, then it is also accurate for nonuniform tracks if the linear density does not increase with depth. The fact that the theory does not apply to tracks having a linear density that increases with depth is a weakness of the present analysis because variations in the track density will change quantitative results. However, the conclusion, that the uniform approximation is reasonably accurate except under some extreme conditions, may still be valid even though the mathematical derivation of the conclusion is not. This is suggested by data presented in [3]. Ions used for the charge collection measurements were alpha particles having an LET that increases with depth. As discussed in more detail in Section V, the RC distribution is expected to be very nonuniform, yet the uniform approximation fit measured data very well.

III. Terminology and Statement of the Mathematical Problem

The assumed physical arrangement consists of a device substrate containing an ion track which extends from the upper substrate surface to a depth equal to the track length. The upper substrate boundary is assumed to be an infinite plane which is a sink for minority carriers. The substrate is assumed to be infinitely thick, although the analysis in the Appendix also treats finite thicknesses. The track radius is arbitrary because an integration in the lateral coordinates eliminates the track radius from the equations. The linear track density is assumed to be uniform over the track length, so the track is completely described by two parameters, which are the track length and the linear density. Charge that reaches the upper surface via diffusion from the track is determined by the two track parameters and by the substrate diffusion length function which describes recombination losses in the substrate. The diffusion length function is assumed to be laterally uniform, but may be highly nonuniform in the vertical coordinate due to a nonuniform RC density.

The uniform approximation estimates collected charge by assuming some appropriately selected effective (uniform) diffusion length. The objective is to use diffusion theory to show that this approximation can, sometimes, provide a reasonably accurate estimate, even when the actual diffusion length function is highly nonuniform. The uniform approximation can obviously produce correct results if different effective diffusion lengths may be assumed for different ion tracks. The objective is to show that reasonably accurate estimates can be obtained for any ion track when the same effective diffusion length is assumed for all track lengths. Let $Q(z)$ denote collected charge when the track length is z. The objective is to show that there is an effective diffusion length having the property that the uniform approximation produces a reasonably accurate estimate of $Q(z)$ for any z between zero and ∞.

The linear track density implicitly contained in $Q(z)$ is superfluous when investigating the adequacy of the uniform approximation. It is convenient to define a normalized Q, which is denoted $I(z)$ and defined to be $Q(z)$ divided by the linear track density. Note that $I(z)$ has the dimensions of distance. The quantity $I(\infty)$ has a special significance because it has two interpretations. The first interpretation is immediately implied by its definition; it is the normalized charge collected from an infinitely long track. Note that for the special case of a uniform diffusion length, the normalized charge collected from an infinitely long track equals the diffusion length. Therefore, the second interpretation of $I(\infty)$ is an effective diffusion length. It is the value that must be assigned to the effective diffusion length in order for the uniform approximation to correctly predict collected charge from an infinitely long track. In fact, $I(\infty)$ is the effective diffusion length that will be used with the uniform approximation in all discussions to follow. This choice for the effective diffusion length ensures that the uniform approximation will be accurate whenever the track is sufficiently long. However, it is still not clear how long is "sufficiently long," or how good the approximation is when the track is not sufficiently long. These questions are answered in the following sections.

IV. Upper and Lower Bounds

Because $I(\infty)$ has a dual interpretation, it is sometimes a convenient unit for measuring both the dependent variable $I(z)$ and the independent variable z, i.e., it is sometimes convenient to plot the dimensionless parameter $I(z)/I(\infty)$ against the dimensionless parameter $z/I(\infty)$. The first parameter is interpreted as the charge collected from a track of length z divided by the charge collected from the infinitely long track, while the second is interpreted as the track length divided by the effective diffusion length. The uniform approximation is expressed in terms of these dimensionless parameters as

$$\frac{I(z)}{I(\infty)} = 1-\exp\left[-\frac{z}{I(\infty)}\right] \quad \text{(uniform approximation)}. \tag{1}$$

It is shown in the Appendix that, no matter what the actual diffusion length function is, an upper bound for the actual $I(z)/I(\infty)$ is given by

$$\frac{I(z)}{I(\infty)} \le \begin{cases} \dfrac{z}{I(\infty)} - \dfrac{1}{4}\left[\dfrac{z}{I(\infty)}\right]^2, & \text{if } \dfrac{z}{I(\infty)} \le 2 \\ 1, & \text{if } \dfrac{z}{I(\infty)} \ge 2. \end{cases} \tag{2}$$

Unfortunately, there is no universal lower bound, except zero. To obtain a nontrivial lower bound, it is necessary to impose a constraint that limits the diffusion length functions that may be considered. The type of constraint that is convenient from the point of view of the analysis is to stipulate that less than some specified fraction of collected charge may come from depths exceeding some specified multiple of the effective diffusion length. For example, we might consider the diffusion length functions satisfying the constraint

> Less than 10% of the charge collected from the
> infinitely long track is from depths exceeding
> four times the effective diffusion length. (3)

This constraint was arbitrarily selected only for illustration. It will be seen shortly that the lower bound consistent with this constraint does not even roughly agree with the uniform approximation, illustrating that there are some problem cases (at least in theory if not in the real world) in which the uniform approximation fails badly. It is shown in the Appendix that a lower bound for any $I(z)/I(\infty)$ consistent with this constraint is given by

$$\frac{I(z)}{I(\infty)} \ge \begin{cases} \dfrac{0.9}{4}\,\dfrac{z}{I(\infty)}, & \text{if } \dfrac{z}{I(\infty)} \le 4 \\ 0.9, & \text{if } \dfrac{z}{I(\infty)} \ge 4. \end{cases} \tag{4}$$

Plots of the right sides of (1), (2), and (4) are shown in Fig. 1. This paper calls a 20% error "reasonably good," so agreement between the upper bound and the uniform approximation is reasonably good. This implies that agreement between the uniform approximation and any actual curve that is above this approximation must also be reasonably good, because any such curve is bracketed between the uniform approximation and the upper bound. However, agreement

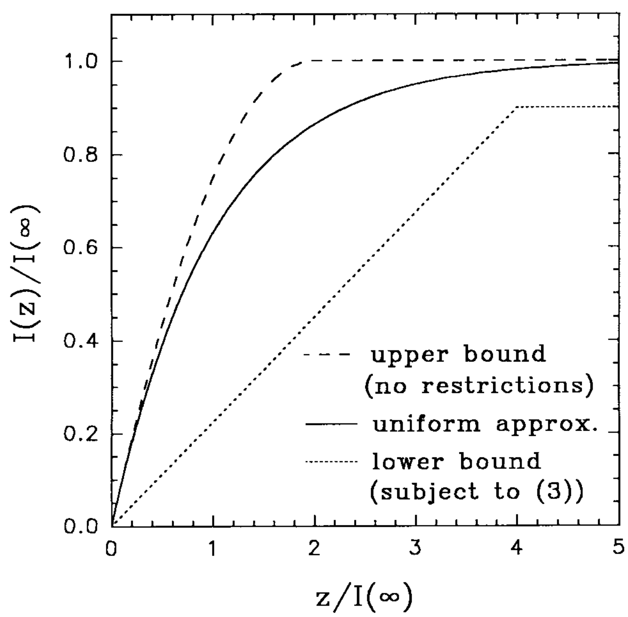


Fig. 1. Plots of the uniform approximation, the upper bound (no restrictions), and the lower bound [subject to (3)].

between the uniform approximation and the lower bound is not as good. If (3) is relaxed to include a larger class of diffusion length functions, the lower bound becomes lower and agreement becomes worse. The uniform approximation fails badly when the actual curve approximates the lower bound corresponding to a constraint that is more relaxed than (3).

It is unfortunate that there are cases such that the uniform approximation does not work well. It can be shown that such cases are produced when a very large number of RC's are confined to a very narrow region that is very close to the upper surface. However, the approximation is reasonably good under all other conditions. For example, if a very large number of RC's are confined to a very narrow region, but this region is at a depth of at least three halves of the effective diffusion length, the actual curve will resemble the upper bound in Fig. 1, which is fairly close to the uniform approximation. The approximation becomes even better for the less extreme cases in which the RC density is spread out to the extent that the density does not vary by more than a factor of a few (e.g., five or less). For these cases, the actual curve will look more like the uniform approximation than either bound shown in Fig. 1. These statements are illustrated by numerical examples in the next section.

V. Numerical Examples

The statements at the end of the previous section can be illustrated by numerical examples. Selection of the examples is motivated by the experimental results presented in [3]. These results actually go beyond validating the applicability of the diffusion equation. They tend to validate the applicability of the diffusion equation combined with the uniform approximation because the uniform approximation was used to derive predictions that were found to fit measured data. Measurements of charge collected by epi SRAM's from ion tracks were compared to predictions that were calculated from the assumption that collected charge consists of the charge liberated in the epi layer plus an additional contribution that diffuses to the epi from the heavily doped substrate below, with the latter charge calculated from the diffusion equation combined with the uniform approximation. Predictions fit the data very well for both virgin devices and devices that had a greatly reduced diffusion length as a result of extensive exposure to heavy ion irradiation. This irradiation was the result of extensive and repeated latchup tests, resulting in a very large accumulated fluence from very heavy ions. The test ions were 295 MeV Kr and 451 MeV Xe, both having a range of just under 40 μm in silicon. Nearly all tests were done at large incident angles (because this was expected to be worst case for latchup) which varied between 60 and 70°, implying ion stopping depths between 14 and 20 μm. All devices were found to have an overlayer thickness of about 4 μm. Different devices had different epi thicknesses, but the device having a 5-μm epi is selected for illustration. The ion stopping depth was 5–11 μm below the bottom of the epi or top of the substrate. Ion energies were strongly dependent on depth for depths between the top of the substrate and stopping depth. It is therefore reasonable to assume that the induced RC distribution in the substrate is confined to the upper 5–11 μm and highly nonuniform within this region.

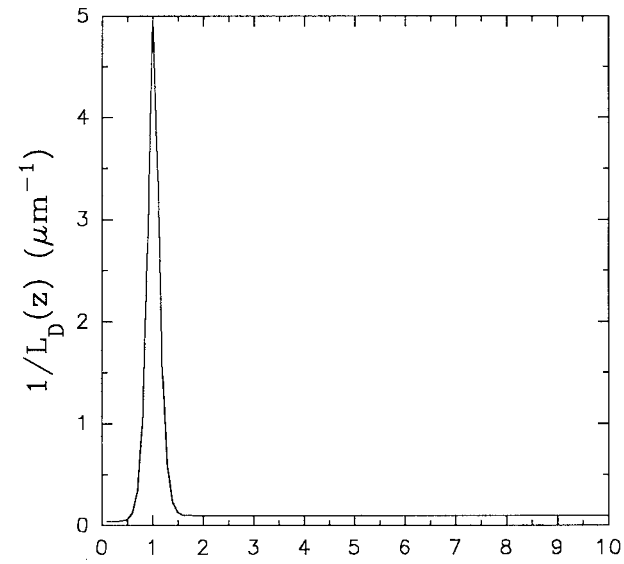
Although [3] presents experimental validation of the uniform approximation, it does not present any theoretical explanation as to why the uniform approximation should apply. The excellent agreement may be somewhat surprising because the actual diffusion length function is likely to be spatially nonuniform as pointed out in the above paragraph; yet, for each device considered, the same effective diffusion length accurately predicts collected charge from any ion track, long or short. The examples below were selected to provide the theoretical explanation, not given in [3], for the excellent agreement reported in [3]. The heavily irradiated devices having reduced diffusion lengths are the most interesting because they are likely to have the greatest nonuniformity in the RC density.

Charge collection measurements found the effective diffusion length to be about 2 μm for the heavily irradiated devices. The virgin devices were found to have an effective diffusion length of about 10 μm. There was sufficient evidence that this difference between diffusion lengths is not due to random part-to-part variations, so it is assumed that the irradiated devices had a 10-μm diffusion length prior to irradiation. Although there is some information regarding the RC distribution, we will enlarge the number of cases considered by ignoring this information and consider a sampling of all possible RC distributions satisfying the constraints that the post-irradiated effective diffusion length is 2 μm and the pre-irradiated effective diffusion length is 10 μm. For each distribution selected from the sampling, we compare the exact $I(z)$ for the selected distribution to the uniform approximation. The reciprocal of the diffusion length function, which is a measure of the RC density, is assumed to be a blip (possibly narrow or possibly broad) representing a (possibly localized or possibly spread-out) RC distribution produced by irradiation damage. The asymptotic (large depth) value of the diffusion length function is assumed to be the pre-irradiated value. The mathematical form of the diffusion length function, which was selected primarily on the basis of analytical tractability, is deferred to the Appendix because the qualitative characteristics shown in the figures discussed later are probably more relevant than mathematical expressions.

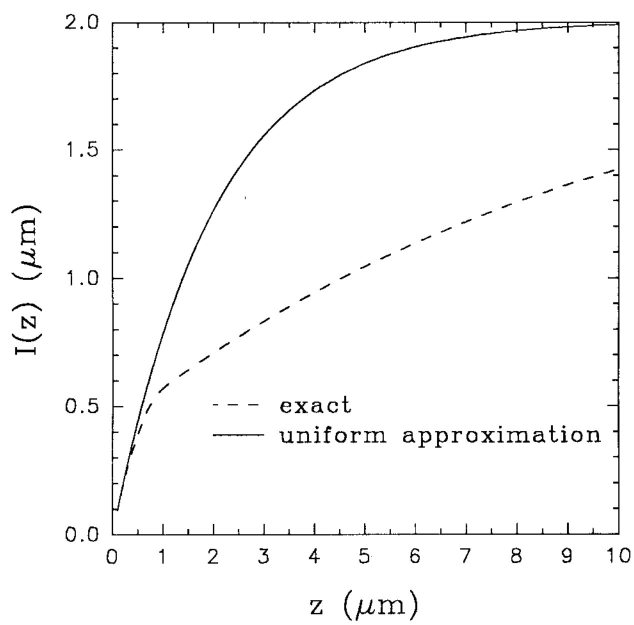
We consider a sampling of all possible diffusion length functions which are described by the equations in the Appendix, have the asymptotic value of 10 μm, and are consistent with $I(\infty) = 2$ μm. The sampling will be worst case from the point of view of demonstrating adequacy of the uniform approximation. This is accomplished by making the blip width as narrow as possible (within limits stated below), to obtain the greatest possible nonuniformity consistent with the stated conditions. Depending on the location of the blip center, an arbitrarily narrow blip may be mathematically compatible with the stated conditions if the blip amplitude is correspondingly large. This occurs when the blip is sufficiently close to the surface. When this is the case, the blip width is taken to be about 1 μm because this is sufficiently close to the mathematical limit (the blip approximates a Dirac delta function). However, if the blip center is sufficiently deep, it is no longer true that an arbitrarily narrow blip can satisfy the condition that $I(\infty) = 2$ μm. Some spread is required so that the RC density extends to higher locations. When this is the case, the blip width is selected to be the smallest value such that the condition can be satisfied. It can be shown that the demarcation between these cases occurs when the blip center depth is approximately twice the effective diffusion length (or 4 μm for the examples to follow). The approximation is accurate when recombination from the RC's outside and above the blip can be neglected. The sampling will use a 1-μm blip width (approximately) if the blip center is at a depth less than 4 μm and the minimum allowed blip width for larger depths.

Note that the shallow blips could be excluded on the basis of relevancy to the measured data, because a shallow and narrow blip could only be produced by damaging ions that stop near the top of the substrate. The damage in the parts considered was produced by ions that stop deeper in the device. Excluding the narrow and shallow blips will give a much more favorable impression regarding the adequacy of the uniform approximation. However, such cases may be relevant under other circumstances, and it may be important to know that the uniform approximation does not always work well. Therefore, the sampling includes such cases even though they are not relevant to the measured data. As a reminder, the RC distributions assumed in the sampling are not intended to be physically realistic; they are intended to be worst case in the sense of having the greatest possible nonuniformity allowed by the constraints.

The sampling is shown in Figs. 2–6. Each figure presents an assumed RC distribution, measured in terms of the reciprocal of the diffusion length function L_D, and compares different predictions of the normalized collected charge I. The z in $L_D(z)$ is interpreted as depth, while the z in $I(z)$ is interpreted as track length. Each I curve identified as "exact" was calculated from the exact equation describing the assumed L_D function.

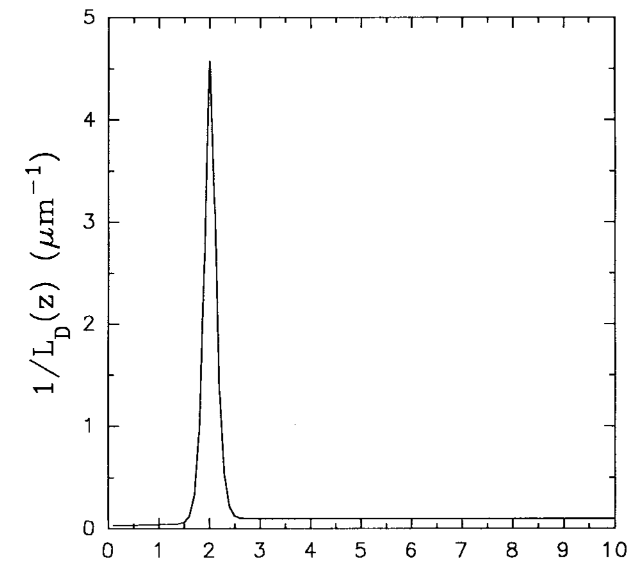


(a)

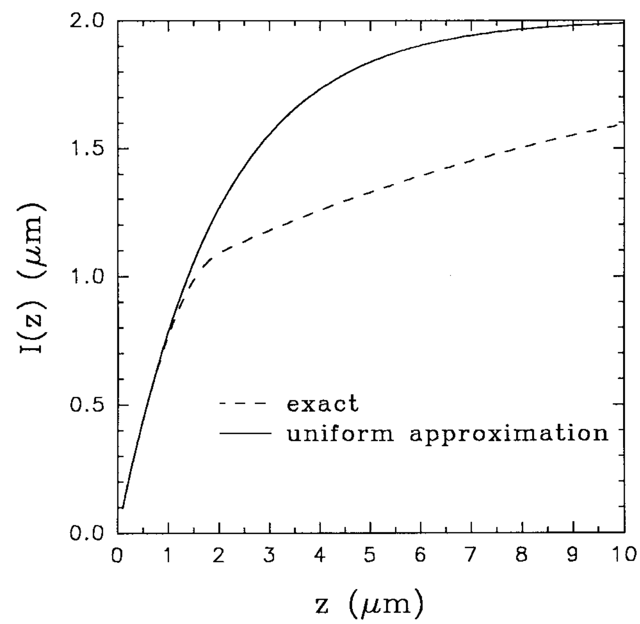


(b)

Fig. 2. The assumed L_D in (a) produces the exact I in (b), which is compared to the uniform approximation in (b).



(a)



(b)

Fig. 3. The assumed L_D in (a) produces the exact I in (b), which is compared to the uniform approximation in (b).

The exact curve in Fig. 2 resembles the uniform approximation when z is less than 1 μm because this places the track above the blip. However, the exact curve is approximately linear and far below the uniform approximation for larger z. In fact, the exact curve in Fig. 2 is even lower, at the larger values of z, than the lower bound shown in Fig. 1. This is because the exact curve in Fig. 2 violates the constraint (3) that applies to the lower bound in Fig. 1. If the blip is moved higher than the 1-μm depth, the point where the exact curve and uniform approximation diverge in Fig. 2 will move further to the left, and the exact curve will approximate the lower bound corresponding to a constraint that is much more relaxed than (3). Such small blip depths are the problem cases in which the uniform approximation fails badly. However, the approximation becomes better as the blip is moved down. It is still not very good in Fig. 3 but is reasonably good in Figs. 4 and 5. Note that the exact curve in Fig. 5 resembles the upper bound in Fig. 1. The blip in Fig. 5 is as deep as it can be without increasing the blip width. It is impossible for $I(\infty)$ to be as small as 2 μm unless the blip adds some RC's to the region above the 4-μm depth. A blip deeper than 4 μm implies that there must be some spread such as shown in Fig. 6. Now that some spread is present, the uniform approximation becomes quite good. Although there is some spread, the RC density is still very nonuniform. It is therefore rather impressive that the uniform approximation works so well.

If the trend started by Figs. 2–6 is continued beyond Fig. 6, the RC density becomes progressively more uniform and the uniform approximation becomes progressively better. Even

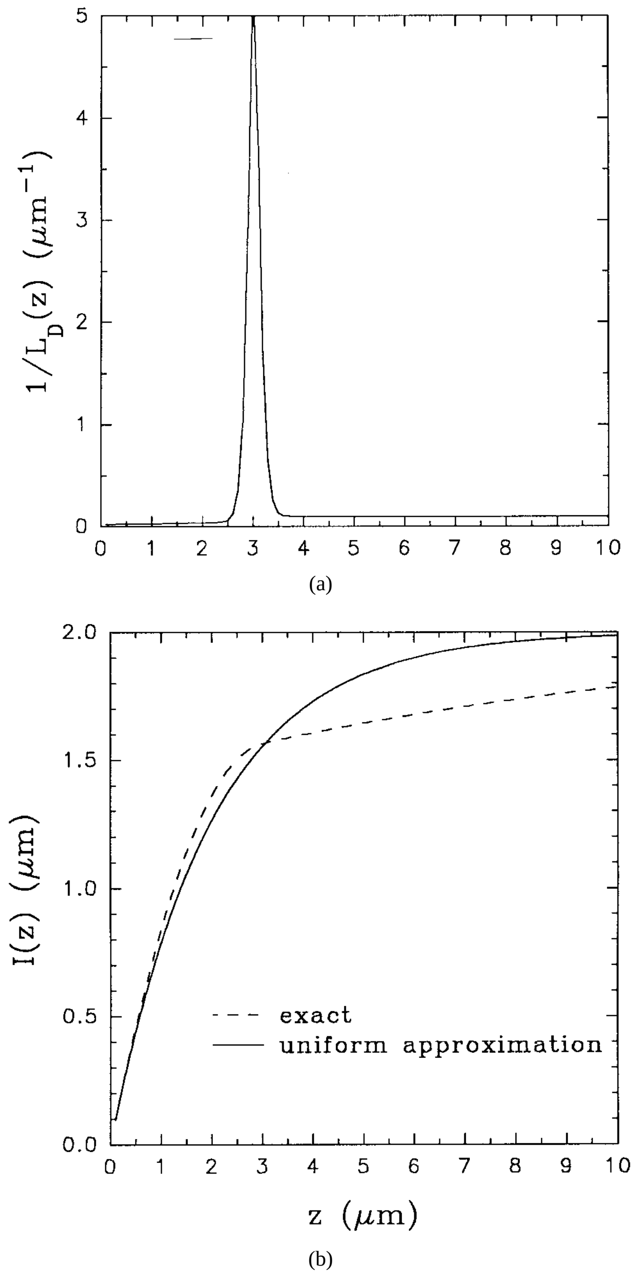


Fig. 4. The assumed L_D in (a) produces the exact I in (b), which is compared to the uniform approximation in (b).

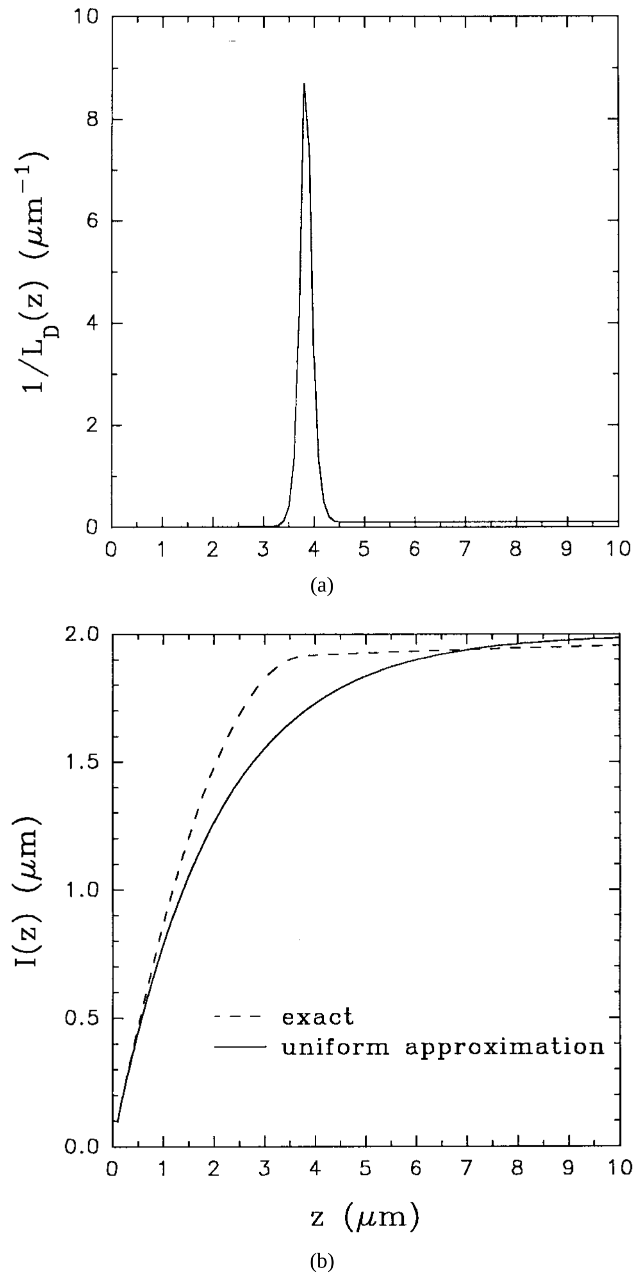


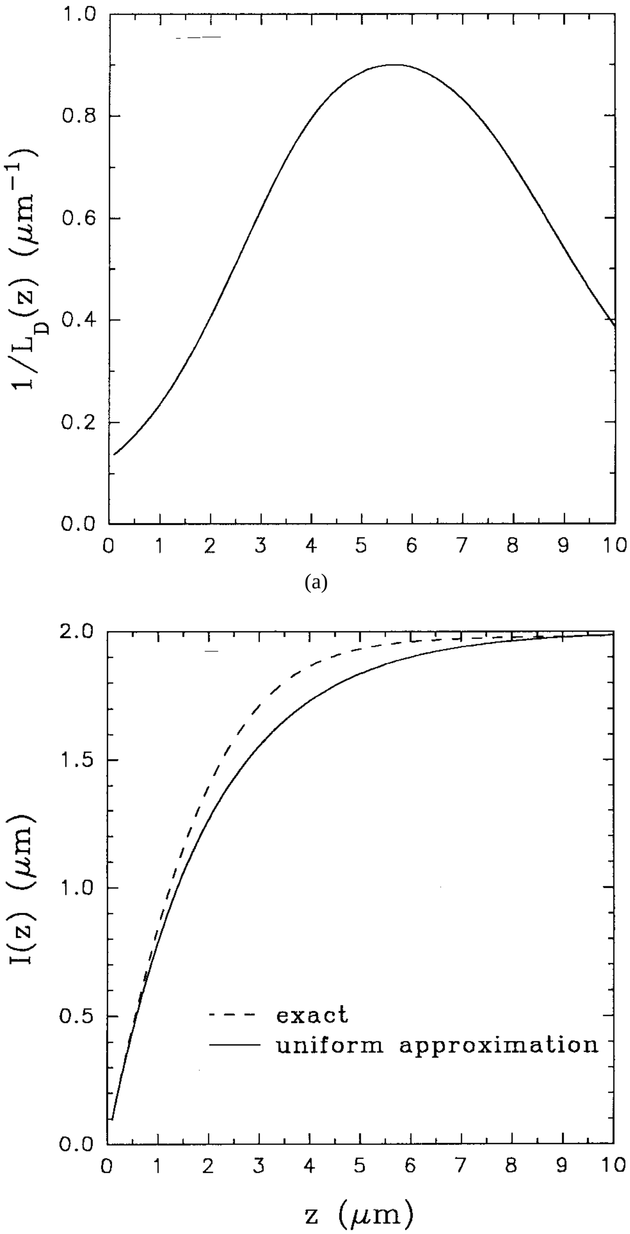
Fig. 5. The assumed L_D in (a) produces the exact I in (b), which is compared to the uniform approximation in (b).

Fig. 6 probably did not carry the trend far enough to represent the actual test devices because the minimum ion stopping depth was 5 μm below the top of the substrate, and most ions stopped deeper. The uniform approximation is probably even more accurate for the RC distribution in the actual devices than indicated in Fig. 6. This is the theoretical explanation for the excellent agreement empirically found and reported in [3].

VI. Conclusions

Accuracy of the uniform approximation was investigated for those cases in which the diffusion equation applies. It was found that, at least for some hypothetical cases, the approximation does not always work well. It fails badly when a large number of RC's are confined to a narrow region at a depth less than 3/2 of the effective diffusion length, as illustrated in Figs. 2 and 3. However, when such extreme cases are excluded, charge collection is insensitive to spatial variations in the RC distribution (subject to the important qualification that all distributions being compared produce the same effective diffusion length), and the uniform approximation ranges from reasonably good to excellent even when the RC distribution is very nonuniform, as illustrated in Figs. 4–6.

The theory does not apply to charge collection from ion tracks having an LET that increases with depth, and this is a limitation of the present work. However, experimental data suggest that the conclusion, that the uniform approximation is reasonably accurate except under some extreme conditions,



(b)

Fig. 6. The assumed L_D in (a) produces the exact I in (b), which is compared to the uniform approximation in (b).

may still be valid even when the mathematical derivation is not.

APPENDIX
MATHEMATICAL ANALYSIS

A. An Expression for $I(z)$

We eventually consider substrates that are effectively infinitely thick, but it is convenient to start with a finite thickness L and take a limit later. The substrate is imagined to lie between two infinite planes which are both sinks for minority carriers. It was shown [11] that the amount of charge Q reaching the upper plane via diffusion from an ion track can be calculated from

$$Q = \int_0^L \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} P_I(\zeta_1, \zeta_2, \zeta)\Omega(\zeta_1, \zeta_2, \zeta)\, d\zeta_1\, d\zeta_2\, d\zeta$$

where P_I is the initial track density (charge per unit volume) and Ω is the charge-collection efficiency. The two coordinates ζ_1 and ζ_2 are lateral coordinates, while ζ is the longitudinal coordinate. The charge collection efficiency is calculated by solving the boundary value problem

$$\nabla^2 \Omega = f^2 \Omega \quad \text{in substrate}, \qquad \Omega = 1 \quad \text{on upper plane}, \qquad \Omega = 0 \quad \text{on lower plane}$$

where f is the reciprocal of the diffusion length function. It is assumed that f depends only on the longitudinal coordinate, so Ω depends only on the longitudinal coordinate. Suppressing the superfluous coordinates, the equation for Ω becomes

$$\frac{d^2\,\Omega(\zeta)}{d\zeta^2} = f^2(\zeta)\,\Omega(\zeta) \text{ for } 0 < \zeta < L, \qquad \Omega(0) = 1, \qquad \Omega(L) = 0. \tag{A1}$$

Integrating the equation for Q with respect to the lateral coordinates gives

$$Q = \int_0^L \rho(\zeta)\,\Omega(\zeta)\, d\zeta$$

where ρ is the linear track density (charge per unit length). We consider the case where this density is uniform over a track length $z \le L$ so that ρ is a step function. It is zero when $\zeta > z$, and constant when $\zeta \le z$. Dividing Q by this constant produces I which is calculated from

$$I(z) = \int_0^z \Omega(\zeta)\, d\zeta. \tag{A2}$$

B. An Upper Bound

When f is specified and Ω is known from (A1), $I(z)$ can be calculated from (A2). However, the present objective is to obtain a bound for the ratio $I(z)/I(L)$ which can be derived when f is not specified and Ω is not known. Such a bound can be obtained by replacing the unknown Ω on the right side of (A2) with an expression that still contains the unknown Ω but has some properties of Ω built into it so that information can be extracted without requiring that Ω be solved. Such an expression can be obtained by converting (A1) into an integral equation. Integrating (A1) twice and then using an integration by parts to change the appearance of the result gives

$$L\,\Omega(z) = L - z - \int_0^L G(z, \zeta) f^2(\zeta)\,\Omega(\zeta)\, d\zeta \tag{A3}$$

where the Green's function G is given by

$$G(z, \zeta) = \begin{cases} (L - z)\zeta, & \text{if } 0 \le \zeta \le z \le L \\ (L - \zeta)\, z, & \text{if } 0 \le z \le \zeta \le L. \end{cases}$$

Substituting (A3) into (A2) and changing the order of integration gives

$$LI(z) = (L - z/2)z - \int_0^L H(z, \zeta)\Psi^2(\zeta)\, d\zeta \tag{A4}$$

where H and Ψ are defined by

$$H(z,\,\zeta)=\begin{cases}(1/2)(z^2/\zeta)(L-\zeta), & \text{if } 0\le z\le\zeta\le L\\ (L-z/2)z-L\zeta/2, & \text{if } 0\le\zeta\le z\le L\end{cases}$$
$$\Psi^2(\zeta)=\zeta f^2(\zeta)\Omega(\zeta). \tag{A5}$$

Note that (A1) implies that Ω is not negative anywhere, so Ψ^2 is not negative anywhere. Another property of Ψ^2 can be derived by first differentiating (A3) and evaluating the derivative at $z=L$ to get

$$L\,\frac{d\Omega(z)}{dz}\bigg|_{z=L}=\int_0^L \Psi^2(\zeta)\,d\zeta-1. \tag{A6}$$

Note that $\Omega(L)=0$ and Ω is not negative anywhere, so Ω cannot be increasing in any neighborhood of the point $z=L$. Therefore, the derivative on the left side of (A6) cannot be positive. Combining this observation with (A6) gives

$$\int_0^L \Psi^2(\zeta)\,d\zeta\le 1. \tag{A7}$$

Another equation containing Ψ^2 is obtained by evaluating (A4) at $z=L$ while using (A5) to get

$$\int_0^L (L-\zeta)\,\Psi^2(\zeta)\,d\zeta=L-2I(L). \tag{A8}$$

An important property of $H(z,\,\zeta)$, implied by (A5), is the type of curvature that it has when plotted against ζ with z fixed. When $\zeta\in(0,\,z)$, H is linear in ζ. When $\zeta\in(z,\,L)$, H has the curvature illustrated in Fig. 7. The H curve is convex when viewed from the left, so it is bounded below by any tangent line, such as illustrated by the lower dashed line in the figure (the upper dashed line is needed in Section VII-C). Selecting any tangent line, which is tangent to the H curve at ζ = any $A\in[z,\,L]$, we have the bound

$$H(z,\,\zeta)\ge\frac{z^2}{2A^2}\left[L(L-\zeta)-(L-A)^2\right]\quad\text{if } \zeta\in[0,\,L]$$
$$\text{and } A\in[z,\,L] \tag{A9}$$

which applies to any $\zeta\in[0,\,L]$ and any $A\in[z,\,L]$. The right side of (A9) is the equation of the tangent line, regarded as a function of ζ with z fixed. Multiplying (A9) by Ψ^2 and integrating and then using (A4), (A7), and (A8) to substitute for the integrals gives

$$I(z)\le z+z^2\left[\frac{I(L)}{A^2}-\frac{1}{A}\right]\quad\text{if } z\in[0,\,L]\text{ and } A\in[z,\,L] \tag{A10}$$

which applies for any $A\in[z,\,L]$. In particular, (A10) applies when $A=z$, and we obtain the obvious result $I(z)\le I(L)$. A stronger statement can be made when $z\le 2I(L)$ because we can then let $A=2I(L)$ and (A10) becomes

$$I(z)\le z-\frac{z^2}{4I(L)}\quad\text{if } z\le 2I(L).$$

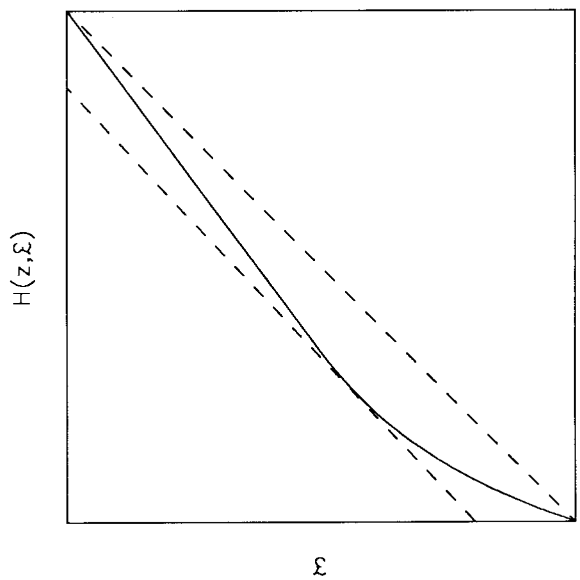


Fig. 7. Illustration of the curvature of $H(z,\zeta)$ when plotted against ζ with z fixed. The H curve (solid) is bounded above by the chord connecting the end points (upper dashed line) and bounded below by any tangent line, such as the lower dashed line shown.

When the above inequality does not apply, we still have $I(z)\le I(L)$, so the bound can be expressed as

$$\frac{I(z)}{I(L)}\le\begin{cases}\dfrac{z}{I(L)}-\dfrac{1}{4}\left[\dfrac{z}{I(L)}\right]^2, & \text{if } \dfrac{z}{I(L)}\le 2\\ 1, & \text{if } \dfrac{z}{I(L)}\ge 2.\end{cases} \tag{A11}$$

The upper bound given by (A11) is the smallest upper bound that applies when no restrictions are imposed on the function f. This can be demonstrated by showing that the bound is approached by a limiting case. To demonstrate this, first note that the left side of (A8) is not negative, so $2I(L)\le L$ (it can be seen from (A1) and (A2) that the equality applies when $f=0$ everywhere). The limiting case occurs when $f^2(\zeta)$ is a Dirac delta function centered at $\zeta=2I(L)$ and with an infinite coefficient selected so that a plot of $\Omega(\zeta)$ [satisfying (A1)] is a straight line connecting the point $(\zeta,\,\Omega)=(0,1)$ to the point $[2I(L),0]$, and with $\Omega(\zeta)=0$ for $\zeta\ge 2I(L)$. A direct evaluation of $I(z)$ from (A2) using this Ω produces the right side of (A11).

The large L limit shown as (2) in Section IV is obtained by simply replacing L with ∞ in (A11).

C. A Lower Bound

The curvature of H discussed in Section VII-B and illustrated in Fig. 7 implies that H is bounded by the chord connecting the end points (the upper dashed line in Fig. 7). This gives the inequality

$$LH(z,\,\zeta)\le z(L-z/2)(L-\zeta)\quad\text{for all } z\in[0,\,L]$$
$$\text{and all } \zeta\in[0,\,L].$$

Multiplying this inequality by Ψ^2, integrating, and then using (A4) and (A8) to substitute for the integrals gives

$$\frac{I(z)}{I(L)} \geq 2\,\frac{z}{L} - \left[\frac{z}{L}\right]^2 \quad \text{for all } z \in [0,\, L]. \tag{A12}$$

The lower bound given by (A12) is the smallest lower bound that applies when no restrictions are imposed on the function f. This can be demonstrated by showing that the bound is approached by a limiting case. Such a case occurs when $f^2(\zeta)$ is a Dirac delta function centered at $\zeta = 0^+$, and with a coefficient selected so that a plot of $\Omega(\zeta)$ [satisfying (A1)] first connects the point $(\zeta, \Omega) = (0, 1)$ to the point $[0^+$, $2I(L)/L]$, and then becomes a straight line connecting the point $[0^+, 2I(L)/L]$ to the point $(L, 0)$. A direct evaluation of $I(z)$ from (A2) using this Ω produces the right side of (A12).

Note that the upper bound in (A11) and the lower bound in (A12) come together when the ratio $L/I(L)$ is close to the smallest allowed value (which is two). This could have been anticipated from the fact that there is only one possible f (zero) which can make $2I(L) = L$, so Ω is completely determined in this limit. Unfortunately, our concern is with the opposite extreme of a large $L/I(L)$. Replacing L with ∞ in (A12) produces a lower bound of zero. To obtain a nontrivial lower bound, it is necessary to impose some constraint that restricts the set of functions from which f may be selected.

The type of constraint that is convenient for analysis is obtained by selecting some depth $Z \in (0, L)$ and some fraction $\alpha \in (0, 1)$ and stipulate that the fractional contribution to $I(L)$, from charge collected from a depth exceeding Z, is not larger than α, i.e.,

$$[I(L) - I(Z)]/I(L) \leq \alpha. \tag{A13}$$

A bound consistent with (A13) can be derived by first evaluating (A4) at $z = Z$ and then use (A13) to eliminate the $I(Z)$ to get

$$\int_0^L H(Z,\, \zeta)\Psi^2(\zeta)\, d\zeta \leq (L - Z/2)Z - L(1-\alpha)I(L). \tag{A14}$$

If $z \geq Z$, we have the obvious bound $I(z) \geq I(Z) \geq (1-\alpha)I(L)$. We now consider the case where $z \leq Z$. Three possibilities to consider are $\zeta \in [0, z]$, $\zeta \in [z, Z]$, and $\zeta \in [Z, L]$. It can be shown from (A5) that all three possibilities result in

$$H(z,\, \zeta) \leq \frac{(L - z/2)z}{(L - Z/2)Z}\, H(Z,\, \zeta)$$

with the equality applying when $\zeta = 0$. Multiplying the above inequality by Ψ^2 and integrating and then using (A4) and (A14) to substitute for the integrals gives

$$I(z) \geq \frac{(L - z/2)z}{(L - Z/2)Z}\, (1-\alpha)I(L) \geq (z/Z)(1-\alpha)I(L), \quad \text{if } 0 \leq z \leq Z.$$

It is convenient to express Z as some multiple γ of $I(L)$. Using this notation, the bounds are expressed as

$$\frac{I(z)}{I(L)} \geq \begin{cases} \dfrac{(1-\alpha)}{\gamma}\, \dfrac{z}{I(L)}, & \text{if } \dfrac{z}{I(L)} \leq \gamma \\ (1-\alpha), & \text{if } \dfrac{z}{I(L)} \geq \gamma \end{cases} \tag{A15}$$

which provides a nontrivial result in the large L limit.

As an example, suppose the set of possible diffusion length functions, that the bound is to apply to, is restricted by the constraint that less than 10% of the charge collected from the infinitely long track is from depths exceeding four times the effective diffusion length. We then let $L = \infty$, $\alpha = 0.1$, $\gamma = 4$, and (A15) becomes (4) in Section IV.

D. Calculations Used for the Examples

The numerical examples in Section V were obtained by solving (A1) with f selected to represent some cases of interest. The easiest way to solve this problem is backward. Instead of selecting an f representing a case of interest and attempting to solve (A1) for Ω, it is easier to select an Ω and use (A1) to find out what f is. This is a trial and error method. If the f produced by a selected Ω does not approximate the function that we would like it to be, it is necessary to try again with another Ω. The Ω used for the numerical examples is given by

$$\Omega(z) = B\, \exp(-kz) + AW\, \ln\left[1 + \exp\left(\frac{z_0 - z}{W}\right)\right] \tag{A16}$$

where k, A, W, and z_0 are constants satisfying

$$k > 0, \qquad 0 < W < 1/k, \qquad A \geq 0, \qquad z_0 \geq 0$$
$$AW\, \ln[1 + \exp(z_0/W)] < 1$$

but are otherwise arbitrary. The constant B is not arbitrary. It is calculated from

$$B = 1 - AW\, \ln[1 + \exp(z_0/W)].$$

The reciprocal of the diffusion length function used in the examples is f calculated from (A1) and (A16). The result is a blip, illustrated in the figures in Section V, which has an asymptotic value equal to k. The blip center depth is approximately z_0 when the blip is narrow. The relationship between blip depth and z_0 is more obscure with wider blips, but the depth increases when z_0 increases. The blip width is most strongly influenced by W and increases when W increases. The blip amplitude is most strongly influenced by A and increases when A increases.

The normalized charge $I(z)$ is calculated from (A2) and (A16). To evaluate the integral in (A2), it is necessary to evaluate the integral S defined by

$$S(X) = \int_0^X \ln[1 + \exp(-t)]\, dt.$$

Special values are given by $S(0) = 0$ and $S(\infty) = \pi^2/12$. The argument X can have either sign. When $X > 0$, S can be evaluated from the series

$$S(X) = \frac{\pi^2}{12} - \sum_{m=1}^{\infty} \frac{(-1)^{m-1}}{m^2} \exp(-mX) \quad \text{if } X \geq 0.$$

The series converges very slowly when $X \approx 0$, although convergence is faster for larger X. Calculations used for the examples approximated the series with the first 100 terms for all $X > 0$. Truncation errors are reduced by adding the terms in reverse order. The series diverges if $X < 0$. This case can be treated by first converting the argument to a positive number using the identity

$$S(X) = -S(-X) - X^2/2$$

and then using the series to evaluate the term containing the positive argument.

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